

IV. *A Proposition relating to the Combination of  
Transparent Lens's with Reflecting Planes.*

By J. Hadley, Esq; V. Pr. R. S. Com-  
municated to the Royal Society, January 9,

1734.

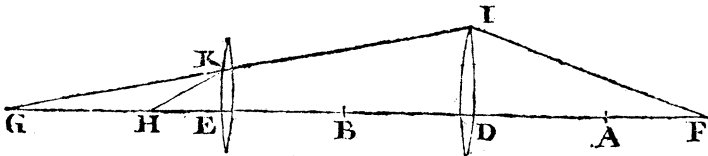
**H**AVING proposed the using of a Telescope with the Instrument for taking Angles, which I formerly laid before this Society, (See N<sup>o</sup> 420.) it gave me Occasion to consider the Effects of the combining several Kinds of Telescopes with reflecting Planes, and, among others, led me to the following Proposition.

That if two Lens's of equal focal Length be put together in the Form of a Telescope, and a Plane Speculum be placed before one of them, so that the Axis of the Telescope make any Angle with its Surface, and a Ray of Light (the Line of whose Direction lies in a Plane perpendicular to that Surface, and passing through the Axis of the Telescope) fall on it, and be reflected from it, so as to pass through the Telescope; then the Line of its last Direction, after passing the Telescope, will make an Angle with that of its first Direction, before its Incidence on the Speculum, very nearly equal to double the Angle made between the Axis of the Telescope, and the Surface of the Speculum.

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## L E M M A

Let the Line  $FG$  be the common Axis of the two Lens's  $ID$  and  $KE$ , of equal focal Lengths ; to which let the Lines  $AD$ ,  $DB$  and  $BE$ , be each equal ; and let a Ray of Light, issuing from a Point in the Axis  $F$ , fall on the Lens  $ID$  at  $I$ , and be there refracted into the Line  $IG$ , cutting the Axis in  $G$ , and meeting the Lens  $KE$  in  $K$ , where let the Ray be again refracted into the Line  $KH$ , cutting the aforesaid Axis in  $H$ : The Angles  $IFD$  and  $KHE$  are very nearly equal.

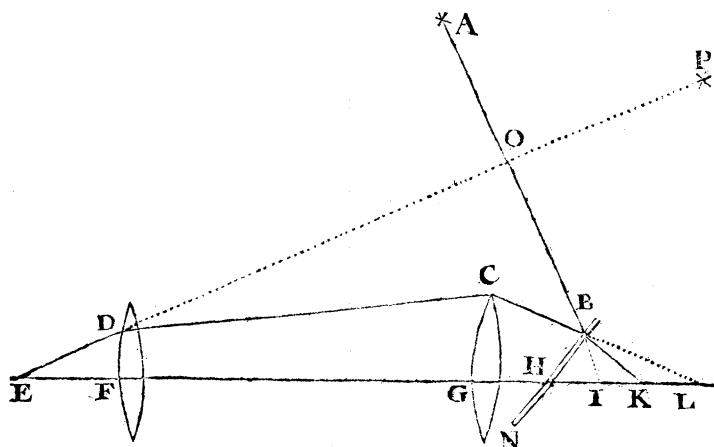


## D E M O N S T R A T I O N .

It is known from *Dioptricks*, that the Lines  $FI$ ,  $IG$ ,  $KH$ , and  $FG$ , are all in the same Plane ; and by the Construction the Lines  $AD$ ,  $DB$ , and  $BE$  are equal ; and by *Propof. 20* of Huygens's *Dioptricks*, the Lines  $FA$ ,  $FD$ , and  $FG$  are continually proportional ; and consequently  $FA$  is to  $AD$  as  $FD$  to  $DG$ , and dividing,  $FA$  is to  $AD$  as  $FD - FA$  ( $=$  to  $AD$ ) is to  $DG - AD$  ( $=$  to  $BG$ .) Therefore  $AD$  is to  $BG$  as  $FD$  to  $DG$ . By the same *Proposition*, the Lines  $BG$ ,  $EG$ , and  $HG$  are also continually proportional, and  $BE$  ( $=$  to  $AD$ ) is to  $BG$  as  $EH$  is to  $EG$ . Hence it follows, that the Lines  $FD$ ,  $DG$ , and  $EH$ ,  $EG$ ,  
are

are Proportionals. But as  $FD$  is to  $DG$ , so is the Tangent of the Angle  $IGD$  or  $KGE$  to the Tangent of the Angle  $IFD$ ; and as  $EH$  is to  $EG$ , so is the Tangent of the Angle  $KGE$  to the Tangent of the Angle  $KHE$ . The Tangent of the Angle  $KGE$  therefore has the same Proportion to the Tangents of each of the Angles  $IFD$  and  $KHE$ , and consequently those Angles are equal. *Q. E. D.*

*N. B.* In the Demonstration of the above-cited Proposition of *Huygens*, the Thickness of the Lens's are neglected, and the Distance of the Points  $I$  and  $K$ , from the Line  $FG$ , supposed very small; so that if either of those are too great, there may arise a sensible Difference between the Angles  $IFD$  and  $KHE$ .



Let  $DF$  and  $CG$  represent the two Lens's put together as before, having their common Axis in the Line  $EL$ , and  $BN$  a plane Speculum to which that Line is inclined in the Angle  $GHN$ , and let

$A B$

A B be a Ray of Light falling on the Speculum at B, as is before expressed, and let it be there reflected towards the Point C of the Lens C G, where it is refracted towards the Point D of the Lens D F, and there again refracted into the Line D E, cutting the Axis in E. The Angle A O P contained between this last Line D E, continued backwards, and the first Line of Incidence of the Ray A B, will be very nearly equal to double the Angle of Inclination of the Axis of the Lens's E L to the Plane of the Speculum B N; *i. e.* double the Angle G H N.

D E M O N S T R A T I O N .

Produce the Lines of Incidence and Reflection of the Ray A B and B C, 'till they meet the Axis of the two Lens's in I and L; and through the Point B draw B K perpendicular to the Plane of the Speculum, and cutting the same Axis in K, the Angles K B L and K B I are equal. The Angle K L B is the Difference of the Angles I K B and K B L; and the Angle H I B is the Sum of the Angles I K B and K B I (equal to K B L): Therefore the Angle I K B is equal to half the Sum of the Angles H I B and K L B. But by the foregoing *Lemma*, the Angles K L B and F E D are very nearly equal. Therefore the Angle I K B is nearly equal to half the Sum of the Angles H I B and F E D; that is, to half the Angle P O B, and its Complement B H I or G H N is nearly equal to half the Angle A O P the Complement of P O B to a Semicircle. Q. E. D.

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If the first Incidence of the Ray be supposed to be in the Line E D, it will proceed in the same Track as before, but with the contrary Directions; so that the Angle E O B made between the first incident Ray and the last reflected, will still be equal to the Double of G H N, as before.

It is evident that on this Principle an Instrument might be constructed, the Effects of which would in a great Measure resemble those of that before mentioned (N<sup>o</sup> 420): But it would be liable to the Errors arising both from the spherical Figure of the Lens's, and also the different Refrangibility of the Rays of Light, when the Object is seen at a Distance from the Axis of the Telescope; altho' those Errors, by a proper Disposition of the Parts of the Instrument, may be reduced to a very small Quantity. However, for this Reason, and also because the Instrument seemed to me to be attended with greater Inconveniencies, both in its Construction and Use, than the other, I have not thought it necessary to give any more particular Description of it.

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V. *An Account of a large Bony Substance found in the Womb, which was shewn to the Royal Society, May the 17th, 1733, by Edward Hody, M. D. F. R. S.*

**E**———S———, aged 57, died in St. James's Market, January, 1725. In examining the *Pelvis* of this Woman, I found a large *Bony-Sub-*

B b Sub-